Calculus Midterm

What is the area $f(x)$ under the line $v(x) = 6 - x$ above the interval from 2 to x? What is the derivative of this $f(x)$?

Answer:

$$6x - \frac{1}{2}x^2 - 10; 6 - x$$

Find two different pairs $f(y)$, $g(x)$ so that $f(g(x)) = \sqrt{1 - x^2}$.

Answer:

$$f(y) = \sqrt{y}, g(x) = 1 - x^2; f(y) = \sqrt{1 - y}, g(x) = x^2$$

Find the fixed point for $F(x) = ax + s$. When is it attracting?

Answer:

$$x^* = \frac{s}{1-a}; |a| < 1$$

If a patient's pulse measures 70, then 80, then 120, what least squares value minimizes $(x - 70)^2 + (x - 80)^2 + (x - 120)^2$? If the patient got nervous, assign 120 a lower weight and minimize $(x - 70)^2 + (x - 80)^2 + \frac{1}{2}(x - 120)^2$.

Answer:

The derivative $2(x - 70) + 2(x - 80) + 2(x - 120)$ is zero at the average $x = \frac{70+80+120}{3} = 90$. Nervous patient: The derivative $2(x - 70) + 2(x - 80) + (x - 120)$ is zero at the weighted average $\frac{2(70)+2(80)+120}{5} = 84$.

$$\lim_{x \to 0} \frac{2x \tan x}{\sin x}$$

Answer:

$$\frac{2x \tan x}{\sin x} = \frac{2x}{\cos x} \to \frac{0}{1} = 0$$

Find numbers A and B so that the straight line $y = x$ fits smoothly with the curve $Y = A + Bx + x^2$ at $x = 1$. Smoothly eans that $y = Y$ and $dy/dx = dY/dx$ at $x = 1$.

Answer:

A = 1, B = -1

If the parabolas $y = x^2 + 1$ and $y = x - x^2$ come closest at $(a, a^2 + 1)$ and $(c, c - c^2)$, set up two equations for a and c.

Answer:

For $y = x^2 + 1$ at $x = a$ and $y = x - x^2$ at $x = c$ we require equal slopes $2a = 1 - 2c$. The normal line $y - (a^2 + 1) = \frac{-1}{2a}(x - a)$ must go through the closest point $y = c - c^2$ at $x = c$. (Compare Problem 23.) Then $(c - c^2) - (a^2 + 1) = \frac{-1}{2a}(c - a)$. (Final solution not required: $c - c^2 - (\frac{1}{2} - c)^2 - 1 = \frac{-1}{1-2c}(c - \frac{1}{2} + c)$ yields a cubic equation for c. Calculus will minimise (distance)^2 which involves x^4. Then derivative = 0 gives the same cubic.)