Five points per problem for a total of 80 points – Show your work to get all the points.

Number 1

Find A and B in $\sqrt{1-x} \approx 1 + Ax + Bx^2$.

Answer: A = ½ B = one eighth (from 3.3 #27)

Number 2

Choose c so that the line y = x is tangent to the parabola y = x2 + C. They have the same slope where they touch.

Answer: The line and parabola have slopes 1and 2x. So the touching point must have x = ½ . There y = ½ for the line, y = $(½)^2$ + c for the parabola, so c = ¼ . (from 2.1 #6)

Number 3

18 Suppose $v(t) = t$ for $t \leqslant 2$ and $v(t) = 2$ for $t \geqslant 2$. Draw the graph of $f(t)$ out to $t = 3$.

Answer: The graph is a parabola f (t) = ft^2 out to f = 2 at t = 2. After that the slope of f stays constant at 2. (from 1.3 #18)

Number 4

What is the area f(x) under the line v(x) = 6 -x above the interval from 2 to x? What is the derivative of this f(x)?

Answer: 6x – ½ x^2 – 10 ; 6 – x (from 5.1 #21)

Number 5

Find g(x) and f(y) and their inverses. z = $(6 + x)^3$

Answer: g(x) = x +6, f(y) = y^3, $g^{-1}(y)$ = y – 6 , $f^{-1}(z) = \sqrt[3]{z}$ (from 4.4 #48)

Number 6

Use the first and third to show pi = 3.14.

12 These four integrals all equal π:

$$\int_0^1 \frac{dx}{\sqrt{x(1-x)}} \quad \int_{-\infty}^{\infty} \frac{\sin x}{x}\,dx \quad \frac{8}{3}\int_0^{\pi} \sin^4 x\,dx \quad \int_0^{\infty} \frac{x^{-1/2}\,dx}{1+x}$$

(a) Apply the midpoint rule to two of them until $\pi \approx 3.1416$.

Answer: They need to work, finding the midpoint, until it shows that pi = 3.14
(From 5.8 #12)

Number 7

$$dy/dx = (y+1)/(x+1), \quad y_0 = 0$$

Answer: x (from 6.5 #5 hadn't done before)

Number 8

Solve the definite integral.

$$\int_0^1 \ln x\,dx$$

Answer: -1 (from 7.1 #27)

Number 9

Solve the difference equation.

18 $y(t+1) = y(t) - 1,\ y_0 = 0$

Answer: y(t) = t (from 6.6 #18)

Number 10

Integrate by parts. Figure if it converges or diverges. You will lose most of the points if you just take a 50/50 guess without showing work.

$$\int_0^e \ln x \, dx \quad \text{(by parts)}$$

Answer: (from 7.5 #9) Only 2 points for the correct answer. Must show some work.

$$x \ln x - x]_0^e = -\infty$$

Number 11 and 12

23 A swimming pool is 4 meters wide, 10 meters long, and 2 meters deep. Find the weight of the water and the total force on the bottom.

24 If the pool in Problem 23 has a shallow end only one meter deep, what fraction of the water is saved? Draw a cross-section (a trapezoid) and show the direction of force on the sides and the sloping bottom.

Answer: (from 8.6 #23-24)

23. (800) (9800) kg

24. The cross-section has length 10 meters and depth 2 meters at one end and 1 meter at the other end. Its area is 10 times 1 ½ = 15 m^2 ; multiply by the width 4m to find the total volume $60m^3$. This is ¾ of the box volume (10)(2)(4) = 80 so ¼ of the volume is saved. The force is perpendicular to the bottom of the pool

Number 13

Find the rectangular coordinates of:

$$(r, \theta) = (3\pi, 3\pi)$$

Answer: (from 9.1 #10 hadn't done before)

$r = 3\pi, \theta = 3\pi$ has rectangular coordinates $\mathbf{x} = -3\pi, \mathbf{y} = 0$

Number 14

Find the answer in the form of y = e^n

$$y''' - y' = 0$$

Answer: (from 9.4 #21)

$$e^t, e^{-t}, e^0$$

Number 15

Find the function that equals the sum of $x + x^3 + x^5 + \dots$

Answer: $\frac{x}{1-x^2}$ (from 10.1 #15)

Number 16

15 Show that the alternating series $1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} + \cdots$ does not converge, by computing the partial sums $s_2, s_4, \dots$.

That's 1 minus one half plus one half minus one fourth plus one third minus one sixth…

Answer: (Answer: 10.3 #15)

Even sums $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots$ diverge; a_n's are not decreasing