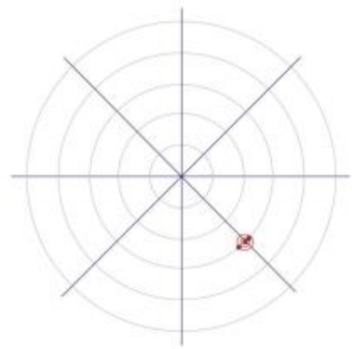
Unit 6 Test

1. $A\left(-3, \frac{3\pi}{4}\right)$



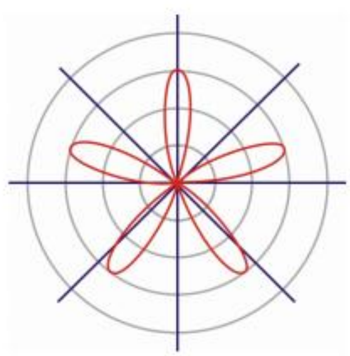
three equivalent coordinates $\rightarrow \left(3, -\frac{\pi}{4}\right), \left(3, \frac{7\pi}{4}\right), \left(-3, -\frac{5\pi}{4}\right)$.

2. $(2, 94°)$ and $(7, -73°)$

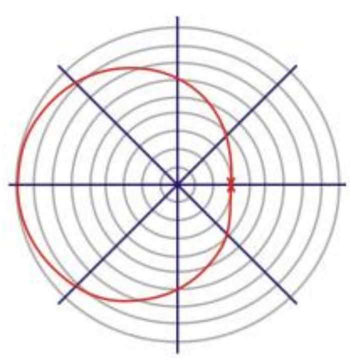
$$\begin{aligned} d &= \sqrt{2^2 + 7^2 - 2(2)(7)\cos(94° - (-73°))} \\ &= \sqrt{4 + 49 - 28\cos 167°} \\ &= \sqrt{80.28} \approx 8.96 \end{aligned}$$

3. Answers:

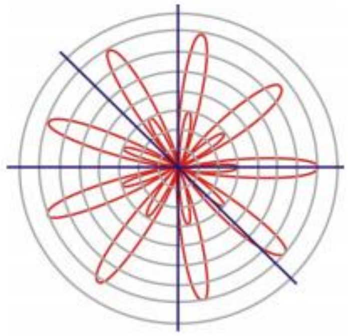
   a. $r = 3\sin 5\theta$



   b. $r = 6 - 3\cos\theta$



c. $r = 2 + 5\cos 9\theta$



4. Answers:

a. $r = 2 - 6\cos\theta$
b. $r = 7 + 3\sin\theta$

5. Answers:

a. $A(-6, 11) \to r = \sqrt{36 + 121} \approx 12.59, \tan\theta = -\frac{11}{6}, \theta = 118.6° \to (12.59, 118.6°)$
b. $B(15, -8) \to r = \sqrt{225 + 64} = 17, \tan\theta = -\frac{8}{15}, \theta = -28.1° \to (17, -28.1°)$
c. $C(9, 40) \to r = \sqrt{91 + 1600} = 41, \tan\theta = \frac{40}{9}, \theta = 77.3° \to (41, 77.3°)$

d.

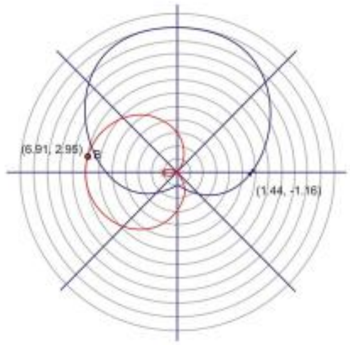
$$\begin{aligned} x^2 + (y-6)^2 &= 36 \\ r^2\cos^2\theta + (r\sin\theta - 6)^2 &= 36 \\ r^2\cos^2\theta + r^2\sin^2\theta - 12r\sin\theta + 36 &= 36 \\ r^2 - 12r\sin\theta &= 0 \text{ or} \\ r^2 &= 12r\sin\theta \\ r &= 12\sin\theta \end{aligned}$$

6. Answers:

a. $D\left(4, -\frac{\pi}{3}\right) \to x = 4\cos\left(-\frac{\pi}{3}\right) = 2, y = 4\sin\left(-\frac{\pi}{3}\right) = -2\sqrt{3} \to (2, -2\sqrt{3})$
b. $E(-2, 135°) \to x = -2\cos 135° = \sqrt{2}, y = -2\sin 135° = -\sqrt{2} \to (\sqrt{2}, -\sqrt{2})$
c. $r = 7 \to r^2 = 49 \to x^2 + y^2 = 49$
d.

$$\begin{aligned} r &= 8\sin\theta \\ r^2 &= 8r\sin\theta \\ x^2 + y^2 &= 8y \\ y^2 - 8y &= -x^2 \\ y^2 - 8y + 16 &= 16 - x^2 \\ (y-4)^2 &= 16 - x^2 \\ x^2 + (y-4)^2 &= 16 \end{aligned}$$

7. $r = 6 + 5\sin\theta$ and $r = 3 - 4\cos\theta$



- angle measures in the graph are in radians
- Note: The two determined points of intersection [(6.91, 2.95) and (1.44, −1.16)] were estimated from the trace function on a graphing calculator and are not precise solutions for either equation.

8. Answer:
   - $-3+8i, x=-3, y=8 \rightarrow r = \sqrt{(-3)^2+8^2} \approx 8.54$
   - $\tan\theta = -\frac{8}{3} \rightarrow \theta = 110.56^\circ$
   - $8.54(\cos 110.56^\circ + i\sin 110.56^\circ)$

9. Answer:
   - $15\angle 240^\circ, r = 15, \theta = 240^\circ$
   - $x = 15\cos 240^\circ = -7.5, y = 15\sin 240^\circ = -\frac{15\sqrt{3}}{2} = -7.5\sqrt{3}$
   - So, $15\angle 240^\circ = -7.5 - 7.5i\sqrt{3}$.

10. Answers:

    a. $\left(7cis\frac{7\pi}{4}\right)\cdot\left(3cis\frac{\pi}{3}\right) = 21cis\left(\frac{7\pi}{4}+\frac{\pi}{3}\right) = 21cis\frac{25\pi}{12}$

    b. $\frac{8\angle 80^\circ}{2\angle -155^\circ} = 4\angle(80^\circ - (-155^\circ)) = 4\angle 235^\circ$

11. $\left[4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^6 = 4^6\left(\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right) = 4096\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

12. -64 in polar form is $64(\cos\pi - i\sin\pi)$

$$[64(\cos(\pi+2\pi k) + i\sin(\pi+2\pi k))]^{\frac{1}{6}}$$

$$2\left(\cos\left(\frac{\pi+2\pi k}{6}\right) + i\sin\left(\frac{\pi+2\pi k}{6}\right)\right)$$

$$2\left(\cos\left(\frac{\pi}{6}+\frac{\pi k}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{\pi k}{3}\right)\right)$$

$$z_1 = 2\left(\cos\left(\frac{\pi}{6}+\frac{0\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{0\pi}{3}\right)\right) = 2\cos\frac{\pi}{6} + 2i\sin\frac{\pi}{6} = \frac{2\sqrt{3}}{2} + \frac{2i}{2} = \sqrt{3}+i$$

$$z_2 = 2\left(\cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right) = 2\cos\frac{\pi}{2} + 2i\sin\frac{\pi}{2} = 2i$$

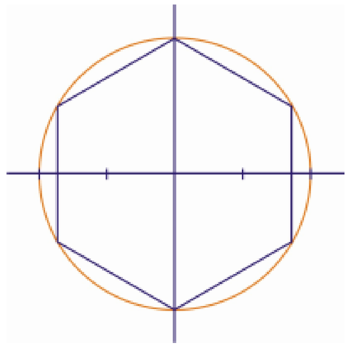
$$z_3 = 2\left(\cos\left(\frac{\pi}{6}+\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)\right) = 2\cos\frac{5\pi}{6} + 2i\sin\frac{5\pi}{6} = -\frac{2\sqrt{3}}{2} + \frac{2i}{2} = -\sqrt{3}+i$$

$$z_4 = 2\left(\cos\left(\frac{\pi}{6}+\pi\right) + i\sin\left(\frac{\pi}{6}+\pi\right)\right) = 2\cos\frac{7\pi}{6} + 2i\sin\frac{7\pi}{6} = -\frac{2\sqrt{3}}{2} - \frac{2i}{2} = -\sqrt{3}-i$$

$$z_5 = 2\left(\cos\left(\frac{\pi}{6}+\frac{4\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{4\pi}{3}\right)\right) = 2\cos\frac{3\pi}{2} + 2i\sin\frac{3\pi}{2} = -2i$$

$$z_6 = 2\left(\cos\left(\frac{\pi}{6}+\frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{5\pi}{3}\right)\right) = 2\cos\frac{11\pi}{6} + 2i\sin\frac{11\pi}{6} = \frac{2\sqrt{3}}{2} - \frac{2i}{2} = \sqrt{3}-i$$

Graph of the solutions:



13. Answer:

$$x^4+32=0 \to x^4=-32+0i=-32(\cos\pi+i\sin\pi)$$

$$[32(\cos(\pi+2\pi k)+i\sin(\pi+2\pi k))]^{\frac{1}{4}}$$

$$2\sqrt[4]{2}\left(\cos\left(\frac{\pi+2\pi k}{4}\right)+i\sin\left(\frac{\pi+2\pi k}{4}\right)\right)$$

$$2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)+i\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)\right)$$

$$z_1=2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)=2\sqrt[4]{2}\cos\frac{\pi}{4}+2i\sqrt[4]{2}\sin\frac{\pi}{4}=\frac{2\sqrt[4]{2}\sqrt{2}}{2}+\frac{2i\sqrt[4]{2}\sqrt{2}}{2}$$
$$=\sqrt[4]{2}^3+i\sqrt[4]{2}^3$$

$$z_2=2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)=2\sqrt[4]{2}\cos\frac{3\pi}{4}+2i\sqrt[4]{2}\sin\frac{3\pi}{4}=-\frac{2\sqrt[4]{2}\sqrt{2}}{2}+\frac{2i\sqrt[4]{2}\sqrt{2}}{2}$$
$$=-\sqrt[4]{2}^3+i\sqrt[4]{2}^3$$

$$z_3=2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{4}+\pi\right)+i\sin\left(\frac{\pi}{4}+\pi\right)\right)=2\sqrt[4]{2}\cos\frac{5\pi}{4}+2i\sqrt[4]{2}\sin\frac{5\pi}{4}=-\frac{2\sqrt[4]{2}\sqrt{2}}{2}-\frac{2i\sqrt[4]{2}\sqrt{2}}{2}$$
$$=-\sqrt[4]{2}^3-i\sqrt[4]{2}^3$$

$$z_4=2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{4}+\frac{3\pi}{2}\right)+i\sin\left(\frac{\pi}{4}+\frac{3\pi}{2}\right)\right)=2\sqrt[4]{2}\cos\frac{7\pi}{4}+2i\sqrt[4]{2}\sin\frac{7\pi}{4}=\frac{2\sqrt[4]{2}\sqrt{2}}{2}-\frac{2i\sqrt[4]{2}\sqrt{2}}{2}$$
$$=\sqrt[4]{2}^3-i\sqrt[4]{2}^3$$