Unit 4 Test

1. Answers:

   a. $-\frac{\pi}{6}$
   b. $\frac{\pi}{6}$
   c. $-\frac{\pi}{3}$
   d. $\frac{3\pi}{4}$
   e. 0
   f. $\frac{\pi}{4}$

2. Answers:

   a. 0.927
   b. 0.461
   c. 1.446
   d. -1.37
   e. 1.792
   f. 0.586

3. Answers:

   a. $\frac{\sqrt{2}}{2}$
   b. 1
   c. 2
   d. $\frac{5}{13}$
   e. $\frac{5}{2\sqrt{6}}$ or $\frac{5\sqrt{6}}{12}$
   f. $\frac{\pi}{3}$

4. Answers:

   a.

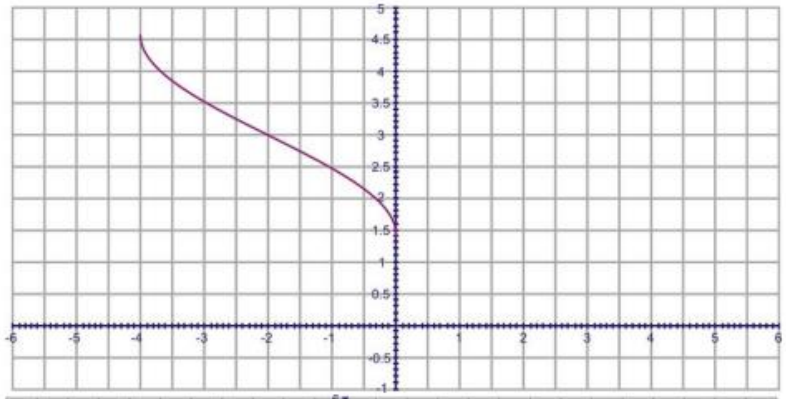
$$\begin{aligned} f(x) &= 5+\cos(2x-1) \\ y &= 5+\cos(2x-1) \\ x &= 5+\cos(2y-1) \\ x-5 &= \cos(2y-1) \\ \cos^{-1}(x-5) &= 2y-1 \\ 1+\cos^{-1}(x-5) &= 2y \\ \frac{1+\cos^{-1}(x-5)}{2} &= y \end{aligned}$$

   b.

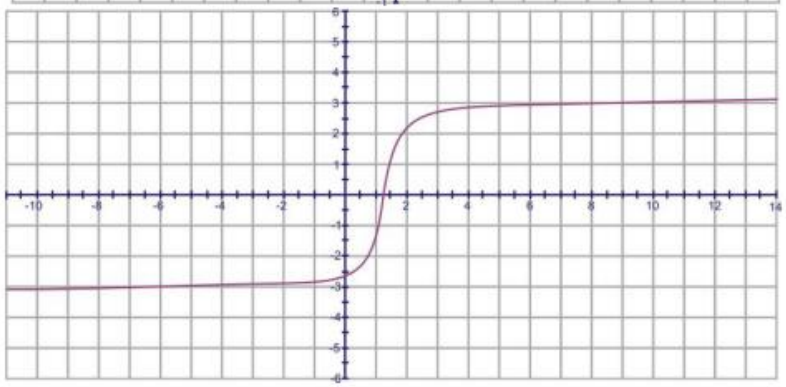
$$\begin{aligned} g(x) &= -4\sin^{-1}(x+3) \\ y &= -4\sin^{-1}(x+3) \\ x &= -4\sin^{-1}(y+3) \\ -\frac{x}{4} &= \sin^{-1}(y+3) \\ \sin\left(-\frac{x}{4}\right) &= y+3 \\ \sin\left(-\frac{x}{4}\right)-3 &= y \end{aligned}$$

5. Answers:

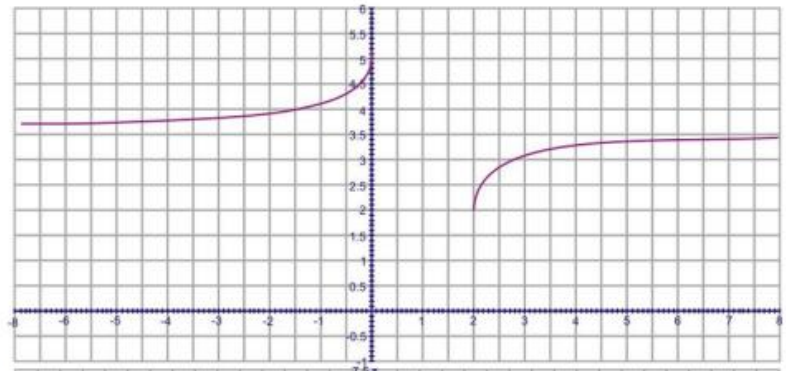
a.



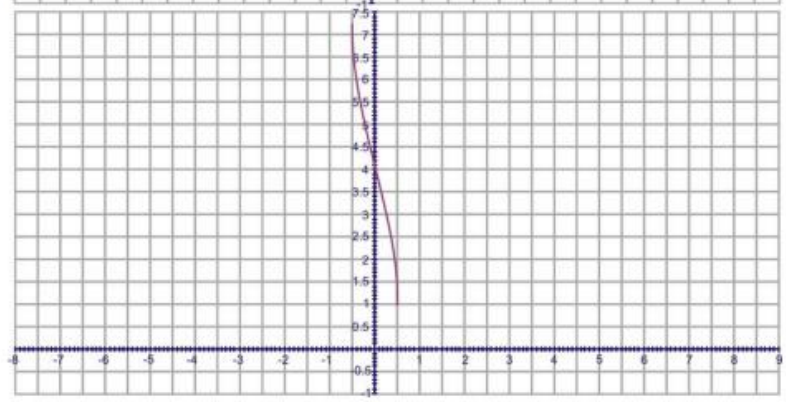
b.



c.



d.



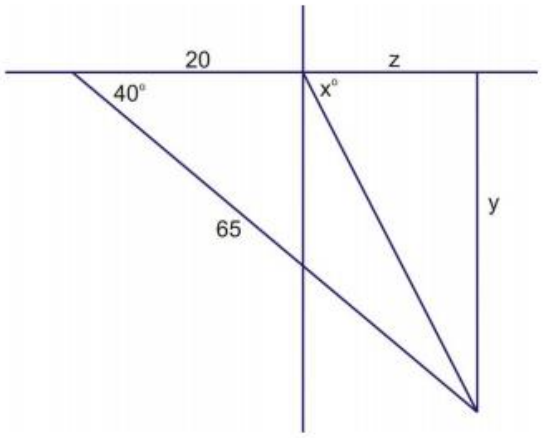
6. Answers:

a. $\sin(\cos^{-1} x^3) = \sqrt{1-(x^3)^2} = \sqrt{1-x^6}$

b. $\tan^2\left(\sin^{-1}\frac{x^2}{3}\right) = \left(\frac{\frac{x^2}{3}}{\sqrt{1-\left(\frac{x^2}{3}\right)^2}}\right)^2 = \frac{\frac{x^4}{9}}{1-\left(\frac{x^4}{9}\right)} = \frac{x^4}{9\left(1-\frac{x^4}{9}\right)} = \frac{x^4}{9-x^4}$

c. $\cos^4(\arctan(2x)^2) = \cos^4(\tan^{-1} 4x^2) = \left(\frac{1}{\sqrt{(4x^2)^2+1}}\right)^4 = \frac{1}{\sqrt{16x^4+1}^4} = \frac{1}{(16x^4+1)^2}$

7. $x°$ can help us find our final answer, but we need to find y and z first.



$$\sin 40° = \frac{y}{65} \rightarrow y = 65\sin 40° = 41.78$$
$$\cos 40° = \frac{20+z}{65} \rightarrow 20+z = 65\cos 40°$$
$$20+z = 49.79 \rightarrow z = 29.79$$
$$\tan x = \frac{41.78}{29.79} \rightarrow x = \tan^{-1}\frac{41.78}{29.79}$$
$$x = 54.51°$$

Now, the bearing from the ship to the point of departure is north, and then $(90-54.51)°$ west, which is written as $N35.49°W$.

8. $36°$ on the 12^{th} of May $= 90° - 36° - 23.5°\cos\left[(132+10)\frac{360}{365}\right] = 72.02°$
9. $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y, a = \sin x$ and $b = \sin y \rightarrow x = \sin^{-1} a$ and $y = \sin^{-1} b$

$$\sin(x \pm y) = a\sqrt{1-\sin^2 y} \pm b\sqrt{1-\sin^2 x}$$
$$\sin(x \pm y) = a\sqrt{1-b^2} \pm b\sqrt{1-a^2}$$
$$x \pm y = \sin^{-1}\left(a\sqrt{1-b^2} \pm b\sqrt{1-a^2}\right)$$
$$\sin^{-1} a \pm \sin^{-1} b = \sin^{-1}\left(a\sqrt{1-b^2} \pm b\sqrt{1-a^2}\right)$$

10. $\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y, a = \cos x$ and $b = \cos y \rightarrow x = \cos^{-1} a$ and $y = \cos^{-1} b$

$$\cos(x \pm y) = ab \mp \sqrt{(1-\cos^2 x)(1-\cos^2 y)}$$
$$\cos(x \pm y) = ab \mp \sqrt{(1-a^2)(1-b^2)}$$
$$x \pm y = \cos^{-1}\left(ab \mp \sqrt{(1-a^2)(1-b^2)}\right)$$
$$\cos^{-1} a \pm \cos^{-1} b = \cos^{-1}\left(ab \mp \sqrt{(1-a^2)(1-b^2)}\right)$$