2.7 Graphing Tangent, Cotangent, Secant, and Cosecant

Test

1. \( 160^\circ \cdot \frac{\pi}{180} = \frac{160\pi}{180} = \frac{8\pi}{9} \)
2. \( \frac{11\pi}{12} \cdot \frac{180^\circ}{\pi} = 11 \cdot 15^\circ = 165^\circ \)
3. \( \cos \frac{3\pi}{4} = \cos 135^\circ = -\frac{\sqrt{2}}{2} \)
4. For \( \tan \theta = \sqrt{3} \), \( \theta \) must equal 60° or 240°. In radians, \( \frac{\pi}{3} \) or \( \frac{4\pi}{3} \).
5. There are many different approaches to the problem. Here is one possibility: First, calculate the area of the red ring as if it went completely around the circle:

\[
A = A_{\text{total}} - A_{\text{gold}}
\]

\[
A = \pi \left( \frac{2}{3} \times 66 \times \frac{1}{2} \right)^2 - \pi \left( \frac{1}{3} \times 66 \times \frac{1}{2} \right)^2
\]

\[
A = \pi \times 22^2 - \pi \times 11^2
\]

\[
A = 484\pi - 121\pi = 363\pi
\]

\[
A \approx 1140.4 \text{ in}^2
\]

Next, calculate the area of the total sector that would form the opening of the “c”

\[
A = \frac{1}{2} r^2 \theta
\]

\[
A = \frac{1}{2} (22)^2 \left( \frac{\pi}{2} \right)
\]

\[
A \approx 190.1 \text{ in}^2
\]

Then, calculate the area of the yellow sector and subtract it from the previous answer.

\[
A = \frac{1}{2} r^2 \theta \rightarrow A = \frac{1}{2} (11)^2 \left( \frac{\pi}{4} \right) \rightarrow A \approx 47.5 \text{ in}^2
\]

\[
190.1 - 47.5 = 142.6 \text{ in}^2
\]

Finally, subtract this answer from the first area calculated. The area is approximately 998 in².
6. Answers:

a. First find the circumference: \(2\pi \cdot 7 = 14\pi\). This will be the distance for the linear velocity. \(v = \frac{dt}{\pi \cdot 7} = \frac{126\pi}{7} \approx 395.84 \text{ cm/sec}\)

b. \(\omega = \frac{\theta}{t} = \frac{\frac{\pi}{3}}{7} \approx 0.698 \text{ rad/sec}\)

7. Given such a quadrilateral, and given that the two transverse angles are identified as equal (i.e., both are marked as \(\theta\) in the picture), the orange segment must be parallel to the opposite (pink) radius segment, and this quadrilateral would have to be a square. This means that \(\theta\) must be equal to 45 degrees, and both the tangent and cotangent of 45 degrees are equal to 1. Also, since the radii of the circle are equal to 1 unit, each of the sides of the quadrilateral (including the cotangent segment) are equal to 1 unit. Therefore, since \(\cot \theta = \frac{x}{y}\), the number of units that is the length of the cotangent segment must be equal to \(\frac{\pi}{4}\).

![Graph of sine and cosine functions]

The intersections are \(\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)\) and \(\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right)\).

9. \(y = -2 + 4 \sin 5x, A = 4, B = 5, p = \frac{\pi}{5}, C = 0, D = -2\)

![Graph of sine function]

10. \(f(x) = \frac{1}{4} \cos \left(\frac{1}{2}(x - \frac{\pi}{2})\right), A = \frac{1}{4}, B = \frac{1}{2}, p = 4\pi, C = \frac{\pi}{2}, D = 0\)

![Graph of cosine function]

11. \(g(x) = 4 + \tan \left(2(x + \frac{\pi}{2})\right), A = 1, B = 2, p = \frac{\pi}{2}, C = -\frac{\pi}{2}, D = 4\)

![Graph of tangent function]
12. \( h(x) = 3 - 6 \cos(\pi x), A = -6, B = \pi, C = 0, D = 3 \)

13. \( y = -1 + \frac{1}{2} \cos 3x \)

14. \( y = \tan 6x \)