6.6 The Product & Quotient Theorems

1. Answers:
   a. $2\angle 56^\circ, 7\angle 113^\circ = (2)(7)\angle(56^\circ + 113^\circ) = 14\angle 169^\circ$
   b. $3(\cos\pi + i\sin\pi), 10\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = (3)(10)cis\left(\pi + \frac{5\pi}{3}\right) = 30cis\frac{8\pi}{3} = 30cis\frac{2\pi}{3}$
   c. $2+3i, -5+11i \rightarrow$ change to polar

$$x = 2, y = 3 \qquad x = -5, y = 11$$
$$r = \sqrt{2^2+3^2} = \sqrt{13} \approx 3.61 \qquad r = \sqrt{(-5)^2+11^2} = \sqrt{146} \approx 12.08$$
$$\tan\theta = \frac{3}{2} \rightarrow \theta = 56.31^\circ \qquad \tan\theta = -\frac{11}{5} \rightarrow \theta = 114.44^\circ$$

$$(3.61)(12.08)\angle(56.31^\circ + 114.44^\circ) = 43.61\angle 170.75^\circ$$

   d. $6-i, -20i \rightarrow$ change to polar

$$x = 6, y = -1 \qquad x = 0, y = -20$$
$$r = \sqrt{6^2+(-1)^2} = \sqrt{37} \approx 6.08 \qquad r = \sqrt{0^2+(-20)^2} = \sqrt{40} = 20$$
$$\tan\theta = -\frac{1}{6} \rightarrow \theta = 350.54^\circ \qquad \tan\theta = \frac{-20}{0} = und \rightarrow \theta = 270^\circ$$

$$(6.08)(20)\angle(350.54^\circ + 270^\circ) = 121.6\angle 620.54^\circ = 121.6\angle 260.54^\circ$$

2. Without changing complex numbers to polar form, you mulitply by FOIL-ing.

$$(2+3i)(-5+11i) = -10+22i-15i+33i^2 = -10-33+7i = -43+7i$$

   The answer is student opinion, but they seem about equal in the degree of difficulty.
3. Answer:

$$\begin{aligned} 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^2 &= 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\cdot 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \\ &= 16\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)\right) \\ &= 16\left(\cos\frac{\pi}{2}i\sin\frac{\pi}{2}\right) \end{aligned}$$

4. Answer:

$$P = (6.80)(7.05)\angle(56.3^\circ - 15.8^\circ), P = 47.9\angle 40.5^\circ watts$$

8. Even though 1 is not a complex number, we can still change it to polar form.

$$1 \rightarrow x = 1, y = 0$$

$$r = \sqrt{1^2+0^2} = 1$$
$$\tan\theta = \frac{0}{1} = 0 \rightarrow \theta = 0^\circ$$

$$\text{So, } \frac{1}{4cis\frac{\pi}{6}} = \frac{1cis0}{4cis\frac{\pi}{6}} = \frac{1}{4}cis\left(0-\frac{\pi}{6}\right) = \frac{1}{4}cis\left(-\frac{\pi}{6}\right).$$