5.6 Vectors

3. When two vectors are summed, the magnitude of the resulting vector is almost always different than the sum of the magnitudes of the two initial vectors. The only times that $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ would be true is when 1) the magnitude of at least one of the two vectors to be added is zero, or 2) both of the vectors to be added have the same direction.
4. Speed (magnitude): $\sqrt{18^2+225^2}=225.7$ and its direction is $\tan\theta=\frac{18}{225}=N4.6°W$.
5. The magnitude is $\sqrt{330^2+410^2}=526.3$ Newtons and the direction is $\tan^{-1}\left(\frac{410}{330}\right)=E51.2°N$.
6. Answers:
   a. $|\vec{a}|=\sqrt{12^2+18^2}=21.6$, direction $=\tan^{-1}\left(\frac{18}{12}\right)=56.3°$.
   b. $|\vec{a}|=\sqrt{(-3)^2+6^2}=6.7$, direction $=\tan^{-1}\left(\frac{6}{-3}\right)=116.6°$.
7. Answers:
   a. $|\vec{a}|=\sqrt{(2-8)^2+(4-6)^2}=6.3$, direction $=\tan^{-1}\left(\frac{4-6}{2-8}\right)=18.4°$.
   b. $|\vec{a}|=\sqrt{(5-3)^2+(-2-1)^2}=3.6$, direction $=\tan^{-1}\left(\frac{-2-1}{5-3}\right)=123.7°$. Note that when you use your calculator to solve for $\tan^{-1}(\frac{-2-1}{5-3})$, you will get $-56.3°$. The calculator produces this answer because the range of the calculator's $y=\tan^{-1}x$ function is limited to $-90°<y<90°$. You need to sketch a draft of the vector to see that its direction when placed in standard position is into the second quadrant (and not the fourth quadrant), and so the correct angle is calculated by moving the angle into the second quadrant through the equation $-56.3°+180°=123.7°$.
8. In both a and b, we have the SAS case, so you can do the Law of Cosines, followed by the Law of Sines.
   a. $(\vec{a}+\vec{b})^2=31^2+31^2-2(31)(31)\cos 132, \vec{a}+\vec{b}=56.6, \frac{\sin 132}{56.6}=\frac{\sin x}{31}, x=24°$
   b. $(\vec{a}+\vec{b})^2=29^2+44^2-2(29)(44)\cos 26, \vec{a}+\vec{b}=22, \frac{\sin x}{44}=\frac{\sin 26}{22}, x=61.3°$