5.4 The Ambiguous Case

5. The answers can be determined as follows:

   a. $a < b\sin A \to \frac{a}{b} < \sin A \to \frac{22}{31} < \sin A \to A > 45.2^\circ$
   b. $a = b\sin A \to \frac{a}{b} = \sin A \to \frac{22}{31} = \sin A \to A = 45.2^\circ$
   c. $a > b\sin A \to \frac{a}{b} > \sin A \to \frac{22}{31} > \sin A \to A < 45.2^\circ$

6. This problem can be done entirely with right triangle trig, but there are several different ways to solve this particular problem.

$$BD = \sqrt{13.7^2 - 9.8^2} = 9.6 \qquad \tan 42.6^\circ = \frac{9.8}{DC} \to DC = 10.7$$
$$\sin 42.6^\circ = \frac{9.8}{AC} \to AC = 14.5 \qquad BC = 9.6 + 10.7 = 20.3$$
$$\sin B = \frac{9.8}{13.7} \to \angle B = 45.7^\circ \qquad \angle A = 180^\circ - 45.7^\circ - 42.6^\circ = 91.7^\circ$$

7. Answers:

   a. $\angle EBD \Rightarrow 7.6^2 = 9.9^2 + 10.2^2 - 2 \cdot 9.9 \cdot 10.2 \cos EBD \Rightarrow 44.4^\circ$
   b. $\angle BDE \Rightarrow \frac{\sin BDE}{9.9} = \frac{\sin 44.4^\circ}{7.6} \Rightarrow 65.7^\circ$
   c. $\angle DEB \Rightarrow 180^\circ - 65.7^\circ - 44.4^\circ \Rightarrow 69.9^\circ$
   d. $\angle BDC \Rightarrow 180^\circ - 65.7^\circ \Rightarrow 114.3^\circ$
   e. $\angle BEA \Rightarrow 180^\circ - 69.9^\circ \Rightarrow 110.1^\circ$
   f. $\angle DBC \Rightarrow 180^\circ - 114.3^\circ - 21.8^\circ \Rightarrow 43.9^\circ$
   g. $\angle ABE \Rightarrow 109.6^\circ - 43.9^\circ - 44.4^\circ \Rightarrow 21.3^\circ$
   h. $\angle BAE \Rightarrow 180^\circ - 21.3^\circ - 110.1^\circ \Rightarrow 48.6^\circ$
   i. $BC \Rightarrow \frac{\sin 114.3^\circ}{BC} = \frac{\sin 21.8^\circ}{10.2} \Rightarrow 25.0$
   j. $AB \Rightarrow \frac{\sin 110.1^\circ}{AB} = \frac{\sin 48.6^\circ}{9.9} \Rightarrow 12.4$
   k. $AE \Rightarrow \frac{\sin 21.3^\circ}{AE} = \frac{\sin 48.6^\circ}{9.9} \Rightarrow 4.8$
   l. $DC \Rightarrow \frac{\sin 43.9^\circ}{DC} = \frac{\sin 21.8^\circ}{9.9} \Rightarrow 19.0$
   m. $AC = 19 + 4.8 + 7.6 = 31.4$

8. We need to find the distance between sensors 2 and 3. If it is less than 6000 ft, then the sensor will be able to detect all motion between the two. First, $4500 > 4000$, so there is going to be one solution.

   - $\frac{\sin S2}{4000} = \frac{\sin 56^\circ}{4500} \to S2 = 47.47^\circ$
   - $S1 + 47.47^\circ + 56^\circ = 180^\circ \to S1 = 76.53^\circ$
   - $\frac{\sin 76.53^\circ}{x} = \frac{\sin 56^\circ}{4500} \to x = 5279$
   - Since $x < 6000$ ft., Sensor 3 will be able to pick up all movement from its location to the location of Sensor 2.

9. The length from $S3$ to $S4$ is x and from $S4$ to $S2$ is y. $180^\circ - 36^\circ - 49^\circ = 95^\circ$, which is the angle at $S4$.

$$\frac{\sin 95^\circ}{5279} = \frac{\sin 36^\circ}{x} = \frac{\sin 49^\circ}{y} \qquad x = 3114.8, y = 3999.3$$