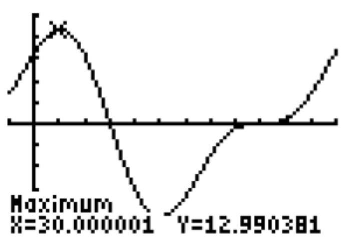
4.4 Applications & Models

1.

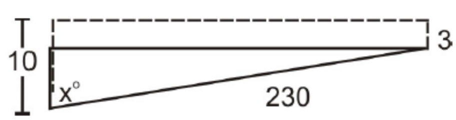
$$\begin{aligned}
I &= I_0 \sin 2\theta \cos 2\theta \\
\frac{I}{I_0} &= \frac{I_0}{I_0} \sin 2\theta \cos 2\theta \\
\frac{I}{I_0} &= \sin 2\theta \cos 2\theta \\
\frac{2I}{I_0} &= 2\sin 2\theta \cos 2\theta \\
\frac{2I}{I_0} &= \sin 4\theta \\
\sin^{-1} \frac{2I}{I_0} &= 4\theta \\
\frac{1}{4}\sin^{-1} \frac{2I}{I_0} &= \theta
\end{aligned}$$

2. The volume is 10 feet times the area of the end. The end consists of two congruent right triangles and one rectangle. The area of each right triangle is $\frac{1}{2}(\sin\theta)(\cos\theta)$ and that of the rectangle is $(1)(\cos\theta)$. This means that the volume can be determined by the function $V(\theta) = 10(\cos\theta + \sin\theta\cos\theta)$, and this function can be graphed as follows to find the maximum volume and the angle θ where it occurs.



Therefore, the maximum volume is approximately 13 cubic feet and occurs when θ is about 30°.

3. See the figure below.



$$\cos x = \frac{7}{230} \rightarrow x = \cos^{-1}\frac{7}{230}$$
$$x = 88.26^\circ$$

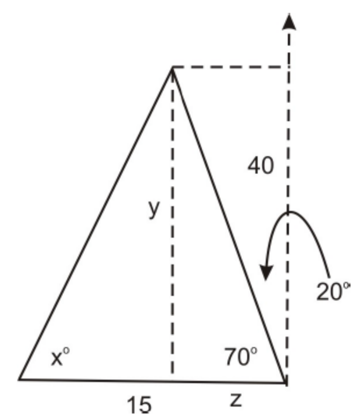
4.

$$i = I_m[\sin(wt+\alpha)\cos\varphi + \cos(wt+\alpha)\sin\varphi]$$
$$\frac{i}{I_m} = \underbrace{\sin(wt+\alpha)\cos\varphi + \cos(wt+\alpha)\sin\varphi}_{\sin(wt+\alpha+\varphi)}$$
$$\frac{i}{I_m} = \sin(wt+\alpha+\varphi)$$
$$\sin^{-1}\frac{i}{I_m} = wt+\alpha+\varphi$$
$$\sin^{-1}\frac{i}{I_m} - \alpha - \varphi = wt$$
$$\frac{1}{w}\left(\sin^{-1}\frac{i}{I_m} - \alpha - \varphi\right) = t$$

5. Answers:

a. 64° on the 16^{th} of November $= 90^\circ - 64^\circ - 23.5^\circ \cos\left[(320+10)\frac{360}{365}\right] = 6.64^\circ$
b. 15° on the 8^{th} of August $= 90^\circ - 15^\circ - 23.5^\circ \cos\left[(220+10)\frac{360}{365}\right] = 91.07^\circ$

6. We need to find y and z before we can find x°.



$$\sin 70^\circ = \frac{y}{40} \rightarrow y = 40\sin 70^\circ = 37.59$$
$$\cos 70^\circ = \frac{z}{40} \rightarrow z = 40\cos 70^\circ = 13.68$$

- Using 15-13.68 as the adjacent side for x, we can now find the missing angle.
- $\tan x^\circ = \frac{37.59}{1.32} = 28.48 \rightarrow x^\circ = \tan^{-1}(28.48) = 87.99^\circ$.
- Therefore, the bearing from the ship back to the point of departure is $W87.99^\circ S$.

7. The maximum displacement for this equation is simply the amplitude, 4.

8. You can use the same picture from Example 5 for this problem.

$$\begin{aligned}
\tan 18^\circ &= \frac{x}{10,000} \\
x &= 10,000 \tan 18^\circ \\
x &= 3249.2 \\
\tan(18^\circ + y) &= \frac{3249.2 + 500}{10,000} \\
\tan(18^\circ + y) &= \frac{3749.2}{10,000} \\
18^\circ + y &= \tan^{-1} \frac{3749.2}{10,000} \\
18^\circ + y &= 20.55^\circ \\
y &= 2.55^\circ
\end{aligned}$$

So, the towers are 2.6° apart.