4.4 Applications & Models

1. 

\[ \frac{I}{I_0} = \frac{I_0}{I_0} \sin 2\theta \cos 2\theta \]
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\[ \frac{I}{I_0} = \sin 2\theta \cos 2\theta \]
\[ \frac{2I}{I_0} = 2 \sin 2\theta \cos 2\theta \]
\[ \frac{2I}{I_0} = \sin 4\theta \]
\[ \frac{1}{2} \sin^{-1} \frac{2I}{I_0} = \theta \]

2. The volume is 10 feet times the area of the end. The end consists of two congruent right triangles and one rectangle. The area of each right triangle is \( \frac{1}{2} (\sin \theta)(\cos \theta) \) and that of the rectangle is \( (1)(\cos \theta) \). This means that the volume can be determined by the function \( V(\theta) = 10(\cos \theta + \sin \theta \cos \theta) \), and this function can be graphed as follows to find the maximum volume and the angle \( \theta \) where it occurs.

Therefore, the maximum volume is approximately 13 cubic feet and occurs when \( \theta \) is about 30°.

3. See the figure below.

\[ \cos x = \frac{7}{230} \rightarrow x = \cos^{-1} \frac{7}{230} \]
\[ x = 88.26^\circ \]
4.

\[ i = I_m \left[ \sin(wt + \alpha) \cos \varphi + \cos(wt + \alpha) \sin \varphi \right] \]
\[ \frac{i}{I_m} = \frac{\sin(wt + \alpha) \cos \varphi + \cos(wt + \alpha) \sin \varphi}{\sin(wt + \alpha + \varphi)} \]
\[ \frac{i}{I_m} = \sin(wt + \alpha + \varphi) \]
\[ \sin^{-1} \frac{i}{I_m} = wt + \alpha + \varphi \]
\[ \sin^{-1} \frac{i}{I_m} - \alpha - \varphi = wt \]
\[ \frac{1}{w} \left( \sin^{-1} \frac{i}{I_m} - \alpha - \varphi \right) = t \]

5. Answers:
   a. 64° on the 16th of November = 90° - 64° - 23.5° \cos \left( \frac{320 + 10}{360} \frac{360}{365} \right) = 6.64°
   b. 15° on the 8th of August = 90° - 15° - 23.5° \cos \left( \frac{220 + 10}{360} \frac{360}{365} \right) = 91.07°

6. We need to find \( y \) and \( z \) before we can find \( x \).

![Diagram](image)

\[ \sin 70° = \frac{y}{40} \rightarrow y = 40 \sin 70° = 37.59 \]
\[ \cos 70° = \frac{z}{40} \rightarrow z = 40 \cos 70° = 13.68 \]

- Using 15-13.68 as the adjacent side for \( x \), we can now find the missing angle.
- \( \tan x° = \frac{37.59}{13.68} = 2.75 \rightarrow x° = \tan^{-1}(2.75) = 78.19° \)
- Therefore, the bearing from the ship back to the point of departure is W87.99°S.

7. The maximum displacement for this equation is simply the amplitude, 4.
8. You can use the same picture from Example 5 for this problem.

\[
\tan 18^\circ = \frac{x}{10,000}
\]

\[
x = 10,000 \tan 18^\circ
\]

\[
x = 3249.2
\]

\[
\tan (18^\circ + y) = \frac{3249.2 + 500}{10,000}
\]

\[
\tan (18^\circ + y) = \frac{3749.2}{10,000}
\]

\[
18^\circ + y = \tan^{-1} \frac{3749.2}{10,000}
\]

\[
18^\circ + y = 20.55^\circ
\]

\[
y = 2.55^\circ
\]

So, the towers are 2.6° apart.