3.7 Products, Sums, Linear Combinations, and Applications

5. (a) If $5\cos x - 5\sin x$, then $A = 5$ and $B = -5$.

   - By the Pythagorean Theorem, $C = 5\sqrt{2}$ and $\cos D = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.
   - So, because B is negative, D is in Quadrant IV.
   - Therefore, $D = \frac{7\pi}{4}$.
   - Our final answer is $5\sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)$.

   (b) If $-15\cos 3x - 8\sin 3x$, then $A = -15$ and $B = -8$.

   - By the Pythagorean Theorem, $C = 17$.
   - Because A and B are both negative, D is in Quadrant III, which means $D = \cos^{-1}\left(\frac{15}{17}\right) = 0.49 + \pi = 3.63$ *rad*.
   - Our final answer is $17\cos 3(x - 3.63)$.

6. Using the sum-to-product formula:

$$\begin{aligned} \sin 11x - \sin 5x &= 0 \\ 2\sin\frac{11x - 5x}{2}\cos\frac{11x + 5x}{2} &= 0 \\ 2\sin 3x\cos 8x &= 0 \\ \sin 3x\cos 8x &= 0 \end{aligned}$$

$\sin 3x = 0$ or $\cos 8x = 0$

$$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi \qquad 8x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$$

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \qquad x = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}, \frac{15\pi}{16}$$

7. Using the sum-to-product formula:

$$\begin{aligned} \cos 4x + \cos 2x &= 0 \\ 2\cos\frac{4x + 2x}{2}\cos\frac{4x - 2x}{2} &= 0 \\ 2\cos 3x\cos x &= 0 \\ \cos 3x\cos x &= 0 \end{aligned}$$

   So, either $\cos 3x = 0$ or $\cos x = 0$

$$\begin{aligned} 3x &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \\ x &= \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \end{aligned}$$

8. Move $\sin 3x$ over to the other side and use the sum-to-product formula:

$$
\begin{aligned}
\sin 5x + \sin x &= \sin 3x \\
\sin 5x - \sin 3x + \sin x &= 0 \\
2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right) + \sin x &= 0 \\
2\cos 4x \sin x + \sin x &= 0 \\
\sin x(2\cos 4x + 1) &= 0
\end{aligned}
$$

So $\sin x = 0$

$$
\begin{aligned}
x = 0, \pi \ \text{ or } \ 2\cos 4x &= -1 \\
\cos 4x &= -\frac{1}{2} \\
4x &= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3} \\
&= \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6} \\
x = 0, &= \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}
\end{aligned}
$$

9. Using the sum-to-product formula:

$$
\begin{aligned}
f(t) &= \sin(200t + \pi) + \sin(200t - \pi) \\
&= 2\sin\left(\frac{(200t+\pi)+(200t-\pi)}{2}\right)\cos\left(\frac{(200t+\pi)-(200t-\pi)}{2}\right) \\
&= 2\sin\left(\frac{400t}{2}\right)\cos\left(\frac{2\pi}{2}\right) \\
&= 2\sin 200t \cos\pi \\
&= 2\sin 200t(-1) \\
&= -2\sin 200t
\end{aligned}
$$